Open questions about q-decreasing words

An n-length binary word is *q-decreasing*, $q \in \mathbb{R}^+$, if every of its length maximal factors of the form $0^a 1^b$ satisfies $a = 0$ or $q \cdot a > b$.

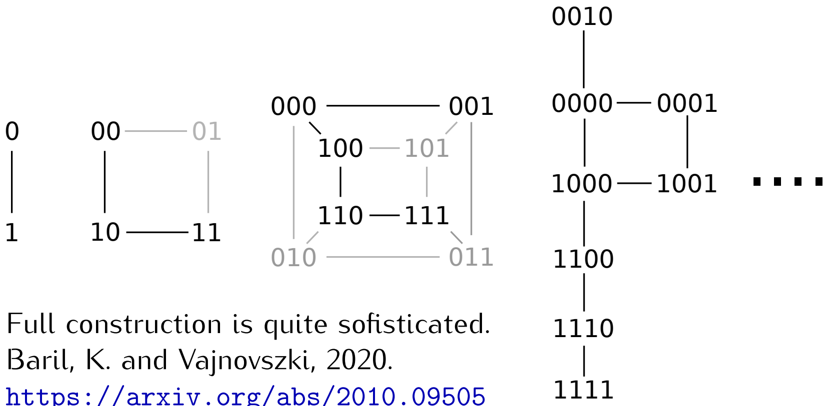
$$\cdots 1 \left| \underbrace{000 \cdots\cdots 00}_{a} \underbrace{111 \cdots 11}_{b} \right| 0 \cdots$$

Let $\mathcal{W}_{q,n}$ be the set of such words of length n

Ex.

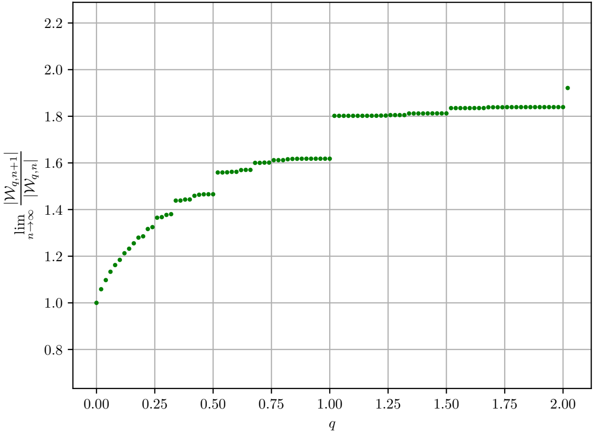
11100101**011**0001 is not 2-decreasing $(2 \cdot 1 \not> 2)$
11100101**011**0001 is 3-decreasing $(3 \cdot 1 > 2)$
11100101**001**0001 is 2-decreasing
111001**01**0010001 is not 1-decreasing

For $q = 1$ a Gray code exists



Full construction is quite sofisticated.
Baril, K. and Vajnovszki, 2020.
https://arxiv.org/abs/2010.09505

Conjecture: there is a Gray code for any $q \in \mathbb{N}^+$



Here is the sequence of positive rational numbers ordered by corresponding jumps of the function $\Phi(q) = \lim_{n\to\infty} |\mathcal{W}_{q,n+1}|/|\mathcal{W}_{q,n}|$

$$1, \frac{1}{2}, 2, \frac{1}{3}, \frac{1}{4}, 3, \frac{2}{3}, \frac{1}{5}, \frac{1}{6}, \frac{3}{2}, \frac{1}{7}, 4, \frac{2}{5}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}, \frac{3}{4}, \frac{1}{11}, \frac{2}{7}, \frac{1}{12}, 5, \frac{3}{5}, \frac{1}{13}, \frac{4}{3}, \frac{1}{14}, \frac{2}{9}, \frac{1}{15}, \frac{1}{16}, ?$$

Next term is ... ???

Combinatorial Gray codes-an updated survey
Torsten Mütze
https://arxiv.org/pdf/2202.01280.pdf
to appear in Electronic Journal of Combinatorics

Gray codes for Fibonacci q-decreasing words.
Jean-Luc Baril, Sk and Vincent Vajnovszki
Theoretical Computer Science, 2022
https://arxiv.org/abs/2010.09505

Fibonacci-run graphs I: Basic properties.
Ömer Eğecioğlu and Vesna Iršič
Discrete Applied Mathematics, 2021
https://arxiv.org/abs/2010.05518

Q-bonacci words and numbers. Sk, Fibonacci conference
https://kirgizov.link/talks/fiboconf.pdf
The Fibonacci Quarterly, 2022
https://arxiv.org/abs/2201.00782

A new paper in preparation with Sergey Dovgal...