# Permuton Limit of 1342-avoiding Permutations

Permutation Patterns 2023


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# Outline

- What's a permuton?

- Structure of 1342-avoiding permutations

- The permuton limit

# Permutons

**Def** Permuton

A permuton is a measure, $\mu$, on $[0,1]^2$ that satisfies

$$\mu\big([a,b]\times[0,1]\big) = \mu\big([0,1]\times[a,b]\big) = b-a$$

# Permutons

—  plot of $\pi = 312$

# Permutons

-  plot of $\pi = 132$

-  corresponding permuton $\mu_\pi$

$$\mu_\pi = 3 \sum_{i=1}^{3} \delta_{[i-1,\, i) \times [\pi(i)-1,\, \pi(i))}$$

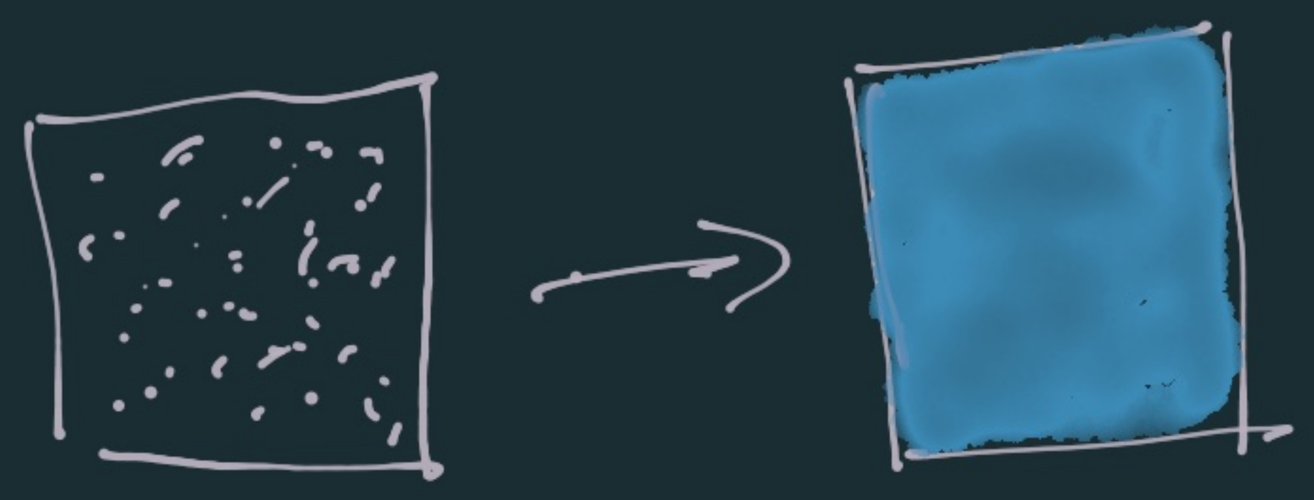
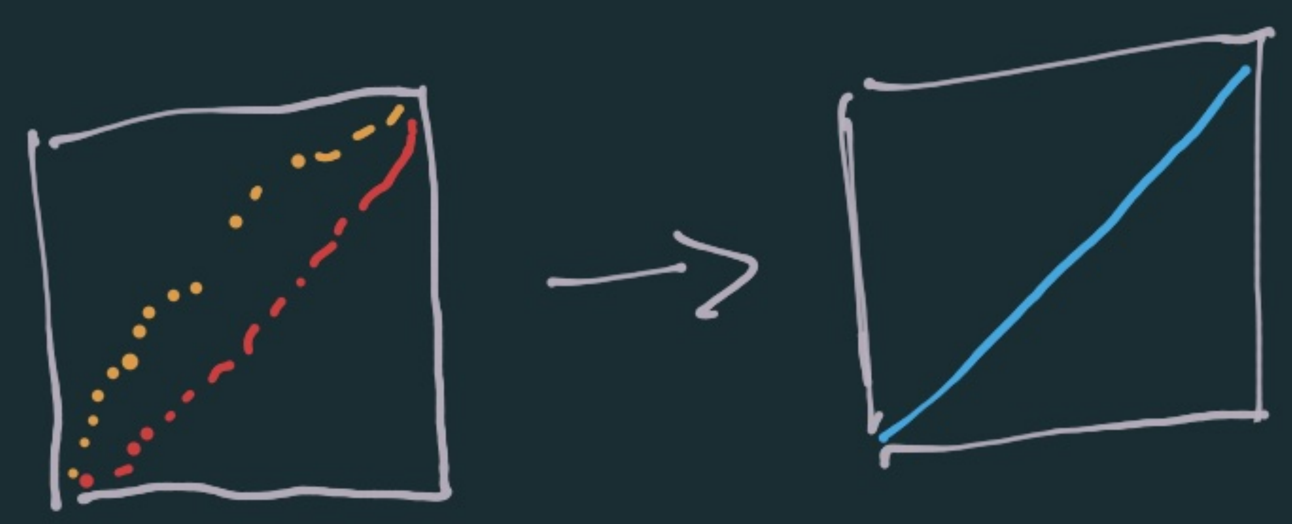
# Permutons

- Let $\pi_n$ be a sequence of permutations.

- Permuton limit of $\pi_n$ is weak limit of $\mu_{\pi_n}$ w/ respect to appropriate topology.

# Permutons

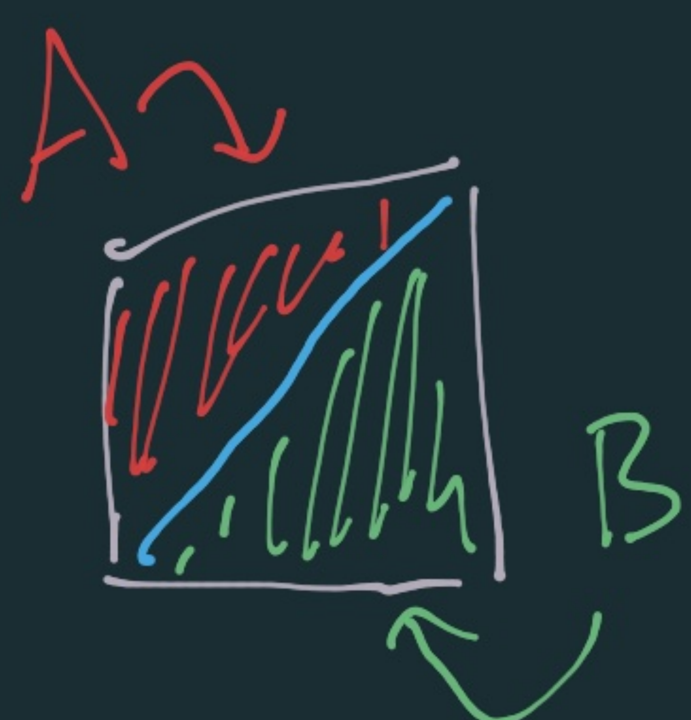
examples

| Permutation | Permuton Limit |
| --- | --- |
| - Uniform in $S_n$ |  |
| - $Av_n(321)$<br>- Miner Pak<br>- Madras Pehlivan |  |

- More recently many others.

# Permutons

**Prop** Let $\mu$ be a permuton

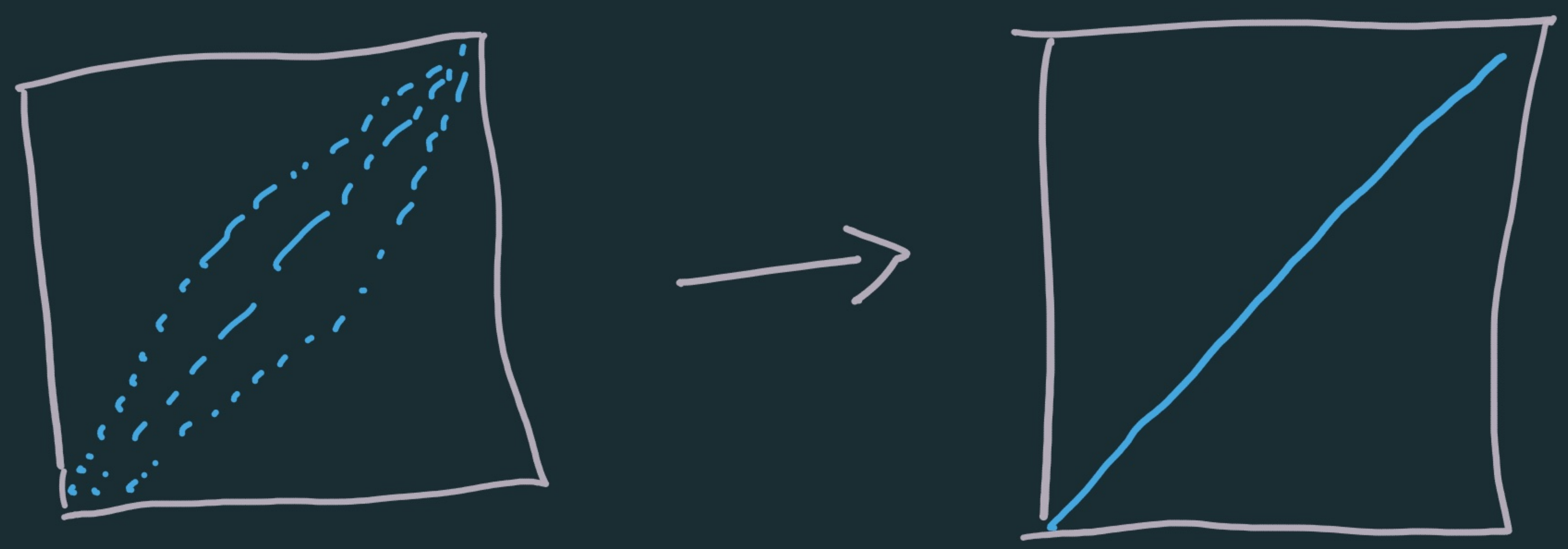



If $\mu(A) = 0$,

then $\mu(B) = 0$,

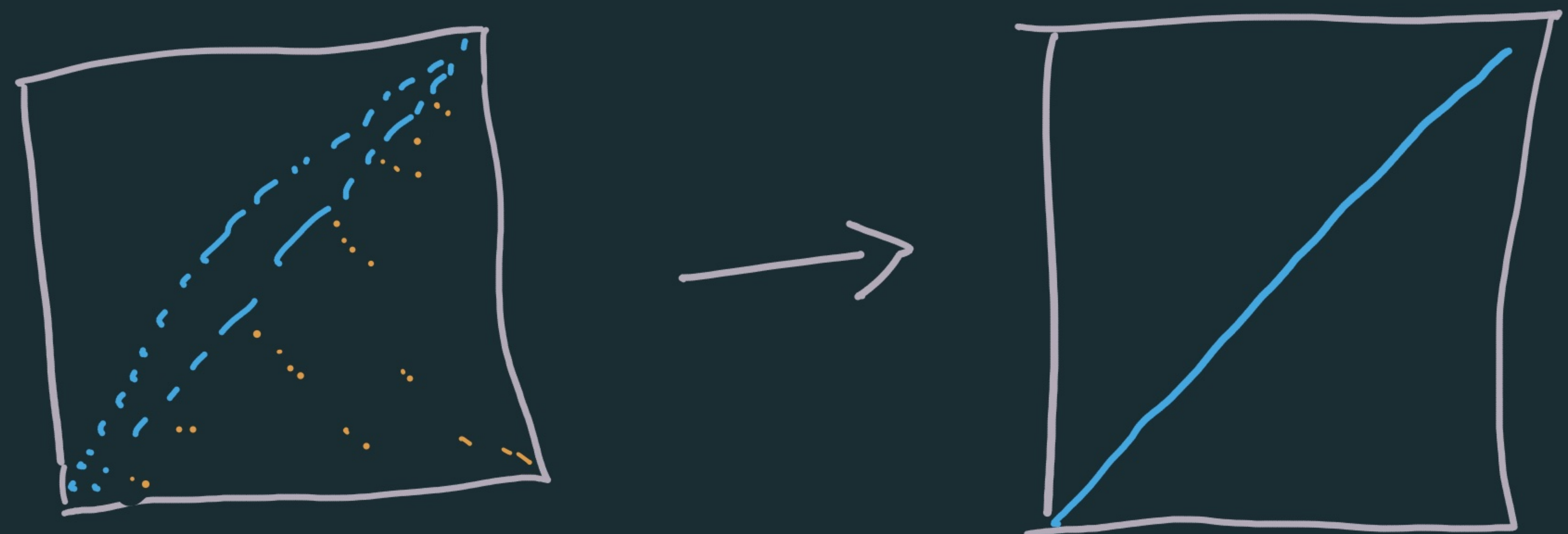
and support of $\mu$ is restricted to the diagonal

# Monotone-avoiders & related results.



- Permuton limit of 4321-avoiders is the diagonal

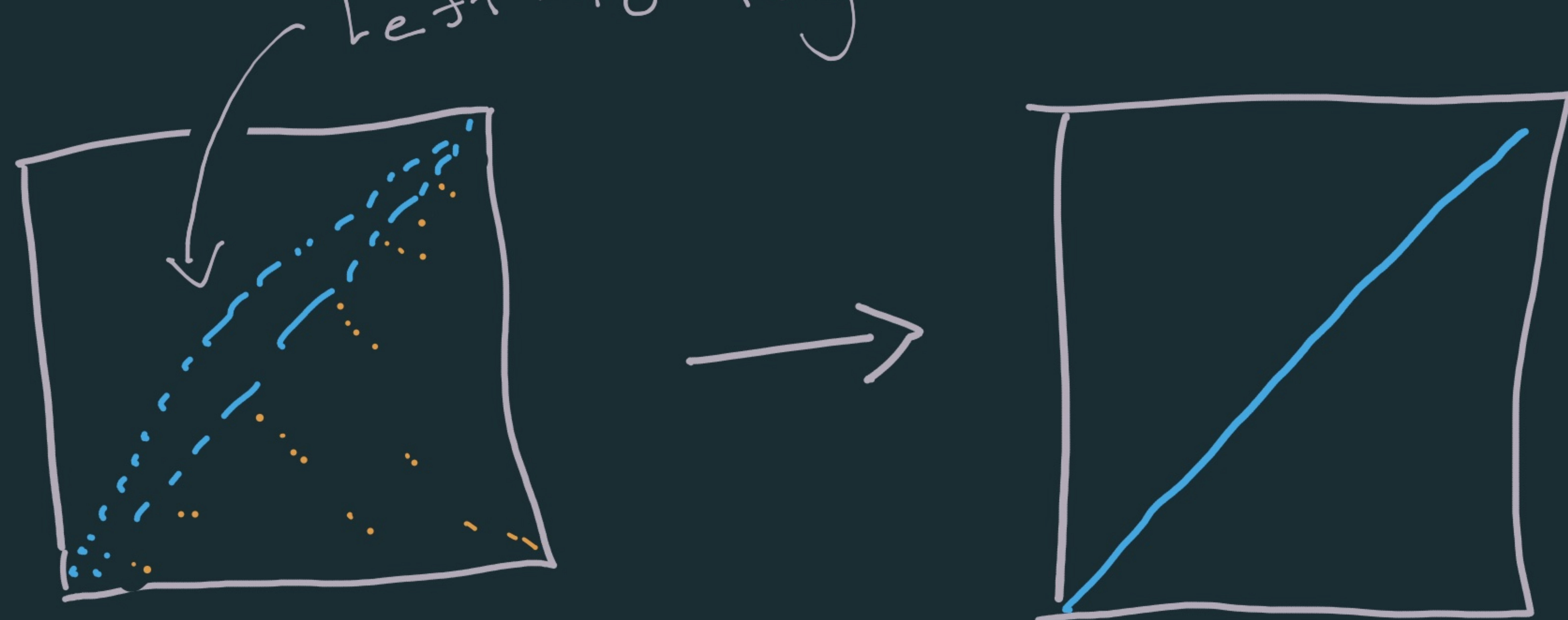
# Monotone-avoiders & related results.



· Permuton limit of 4312-avoiders is the diagonal

# Monotone-avoiders & related results.

Left-to-Right maxima unchanged



via Backelin West & Xin bijection '07

- Permuton limit of 4312-avoiders is the diagonal

# Monotone-avoiders & related results.

Left-to-Right maxima unchanged via Backelin West & Xin bijection '07

- Permuton limit of 4312-avoiders is the diagonal & much more

Apologies for False Claims in Abstract

# 1342-avoiding permutations

- $|Av_n(1342)| \approx 8^n$ (Bóna '97)

- In bijection w/ forest of $\beta(0,1)$ description trees (Cori, Jacquard, Schaeffer)

# 1342-avoiding permutations

**Def** $\beta(0,1)$-tree is a tree $T$, w function $\ell$ that satisfies

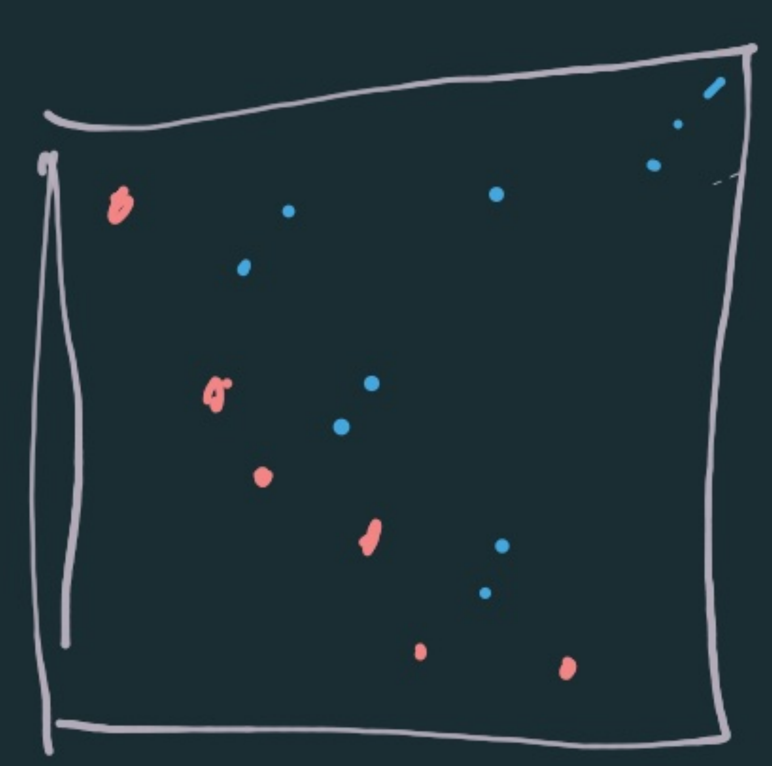
- $\ell(v) = 0$ if $v$ is a leaf
- $\ell(v) \leq 1 + \sum_{w \in \mathcal{C}(v)} \ell(w)$
- $\ell(\text{root}) = \sum_{w \in \mathcal{C}(v)} \ell(w)$

# 1342 - avoiding permutations

- $\mathcal{L}(T)$ set of labellings of $T$

$$\left\{\begin{array}{l} \bullet\ \ell(v) = 0 \text{ if } v \text{ is a leaf} \\ \bullet\ \ell(v) \leq 1 + \sum_{w \in \mathcal{C}(v)} \ell(w) \\ \bullet\ \ell(\text{root}) = \sum_{w \in \mathcal{C}(v)} \ell(w) \end{array}\right.$$

- $|\mathcal{L}(T)| \leq C_{n-1}$ (Catalan number)
- if $\ell(v) = 0$ for all $v$, 132-avoiding
- other labelings on same tree have same LR - minima.



# 1342-avoiding permutations

# 1342-avoiding permutations

- $$|Av_n(1342)| = \sum_{\sigma \in |Av_n(132)|} |\mathcal{L}(T_\sigma)|$$

- $$M_{\pi} = \max_i \left(n+1-(\sigma(i)+i)\right)$$

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**Prop** For $\sigma \in Av_n(132)$

$$P\left( M_\sigma > \delta n \right) < q(n) \cdot e^{-\delta^2 n}$$

Proof: Combine Kaigh ('76) w/ Hoffman, Rizzolo, S. '17

# 1342-avoiding permutations

- $|Av_n(1342)| = \sum_{\sigma \in |Av_n(132)|} |\mathcal{L}(T_\sigma)|$ $\qquad \mathcal{L}(T) < 4^n$

- $M_{\tilde{\pi}} = \max_i \left(n+1-(\sigma(i)+i)\right)$

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**Prop** For $\tau \in Av_n(1342)$

$$P(M_{\tilde{\tau}} > \delta n) < \frac{q(n) \cdot e^{-\delta^2 n} \cdot |Av_n(132)| \cdot 4^n}{|Av_n(1342)|}$$

if $\delta^2 < \log 2$ this goes to $0$.

# 1342-avoiding permutations

Not what I claimed to be true

- $$|Av_n(1342)| = \sum_{\sigma \in |Av_n(132)|} |\mathcal{L}(T_\sigma)|$$

- $\pi$ is bad if LR-min far from anti-diagonal.

$$|BAD| = \sum_{\substack{\sigma \in \text{bad} \\ Av_n(132)}} |\mathcal{L}(T_\sigma)|$$

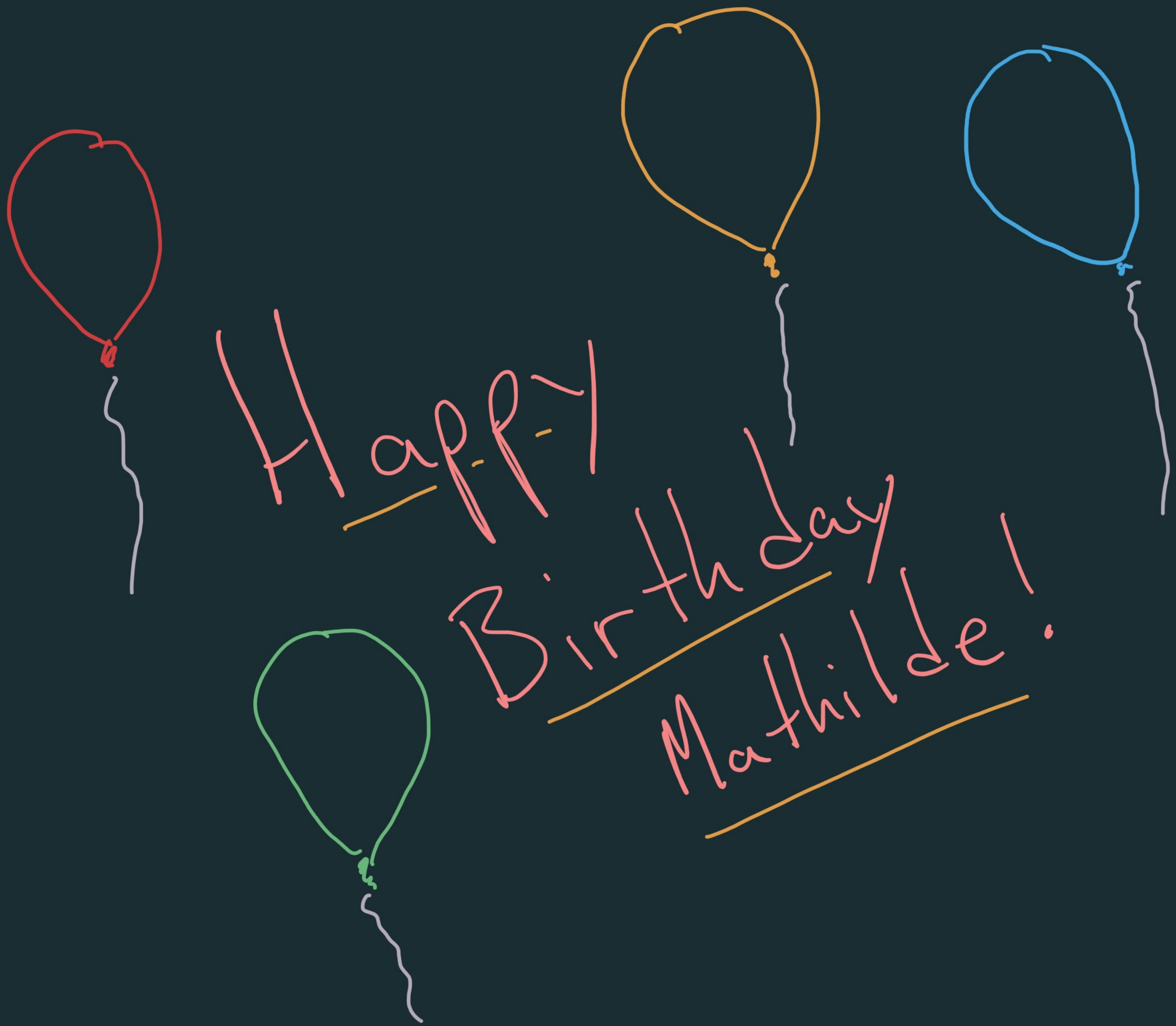
# 1342-avoiding permutations

- $\pi$ is BAD if LR-min far from anti-diagonal.

$$|BAD| = \sum_{\substack{\sigma \in BAD \\ \cap Av_n(132)}} |\mathcal{L}(T_\sigma)|$$

- Show $|BAD| \ll 8^n$
- Can ignore trees w/ height $> \sqrt{\log 2}\, n$
- $|\mathcal{L}(T_\sigma)|$ is usually $\ll 4^n$

need to understand this better!


Happy
Birthday
Mathilde!